第 1 講 The Sign of Gauss Curvature

L1_A 1. Review: Some Setting in the Last Semester
2. Example: The Surface Given by z=h(x,y) with K(p)>0

L1_B 1. Example: The Surface Given by z=h(x,y) with K(p)<0
2. Example: The Surface Given by z=h(x,y) with K(p)=0

L1_C 1. Example: Torus and Monkey Saddle

第 2 講 Geometric Interpretation of Gauss Curvature

L2_A 1. Orientation Preserving and Orientation Reversing Map
2. Proposition: The Gauss Map is Orientation Preserving at Elliptic Point and Orientation Reversing at Hyperbolic Point

L2_B 1. Proposition: Geometric Interpretation of Gauss Curvature

L2_C 1. Proposition: Geometric Interpretation of Gauss Curvature (cont.)
2. Examples: Sphere and Trough-Shaped Surface

第 3 講 Local Convex and Curvature

L3_A 1. Review: Geometric Interpretation of Gauss Curvature
2. Remark: Similar Result for Curvature of Plane Curve
3. Tangent Indicatrix
4. Locally Convex and Strictly Locally Convex

L3_B 1. Note: Relations between Curvature and Locally Convex

L3_C 1. Note: A Critical Point of a Distance Function on a Surface

第 4 講 The Rigidity of the Sphere

L4_A 1. Theorem: A Compact Surface Has an Elliptic Point

L4_B 1. Theorem: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere
2. Lemma: Three Conditions of a Point to Be an Umbilical Point
3. Proof: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere

L4_C 1. Proof: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere (cont.)
2. Proof: Three Conditions of a Point to Be an Umbilical Point

第 5 講 The Rigidity of the Sphere (cont.)

L5_A 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

L5_B 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

L5_C 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

第 6 講 Vector Field

L6_A 1. Vector Field
2. Trajectory of a Vector Field
3. Examples: w=(x,y) and w=(y,-x)

L6_B 1. Theorem: Existence and Uniqueness of the Trajectory of a Vector Field
2. Theorem: Existence of the Local Flow a Vector Field

L6_C 1. Ruled Surface, Ruling and Directrix
2. Examples: Plane, Cylinder, Cone and Hyperboloid of Revolution

第 7 講 Ruled Surface

L7_A 1. Line of Striction

L7_B 1. Condition of a Ruled Surface Given by the Line of Striction as Directrix being a Regular Surface
2. Gauss Curvature of a Ruled Surface Given by the Line of Striction as Directrix

L7_C 1. Developable Surface
2. Developable Surface Has Gauss Curvature Zero at Regular Points

第 8 講 Developable Surface

L8_A 1. Two Subclasses of Developable surface

L8_B 1. Two Subclasses of Developable surface (cont.)
2. Example: The Envelope of the Family of Tangent Planes Along a Curve of a Surface

L8_C 1. The Envelope of the Family of Tangent Planes Along a Curve of a Surface Is Developable

第 9 講 Minimal Surface

L9_A 1. Review: Developable Surface
2. Minimal Surface
3. Normal Variation
4. Interpretation of Minimality

L9_B 1. Interpretation of Minimality (cont.)

L9_C 1. Interpretation of Minimality (cont.)
2. Proposition: A Parametrized Surface Is Minimal if and only if A′(0)=0

第 10 講 Minimal Surface (cont.)

L10_A 1. Proof: A Parametrized Surface Is Minimal if and only if A′(0)=0
2. Isothermal Parametrized Surface
3. Theorem: If x Is an Isothermal Parametrized Surface Then x_uu+x_vv=2(a^2)HN

L10_B 1. Theorem: If x Is an Isothermal Parametrized Surface Then x_uu+x_vv=2(a^2)HN (cont.)
2. Harmonic Function
3. Corollary: An Isothermal Parametrized Surface Is Minimal if and only if Its Coordinate Functions Are Harmonic
4. Introduction: Development of Minimal Surface

L10_C 1. Examples: Catenoid and Helicoid
2. Proposition: Any Minimal Surface of Revolution Is an Open Subset of a Plane or a Catenoid
3. Proposition: Any Ruled Minimal Surface Is an Open Subset of a Plane or a Helicoid
4. Theorem: There Is No Compact Minimal Surface

第 11 講 The Intrinsic Geometry of Surfaces: Isometries and Conformal

L11_A 1. Parametrizations for Catenoid and Helicoid
2. Isometry and Local Isometry
3. Example: Local Isometric; The Cylinder and Plane in R^2

L11_B 1. Example: Local Isometric; The Cylinder and Plane in R^2 (cont.)
2. Example: Every Helicoid Is Locally Isometric to Catenoid
3. Proposition: Two Regular Surfaces Have the Same First Fundamental Form in Domain U if and only if They Are Locally Isometric

L11_C 1. Proposition: Two Regular Surfaces Have the Same First Fundamental Form in Domain U if and only if They Are Locally Isometric (cont.)
2. Conformal

第 12 講 The Intrinsic Geometry of Surfaces: Isometries and Conformal (cont.)

L12_A 1. Review: Isometry and Conformal
2. Note: A Conformal Map Preserves the Angle between Two Tangent Vectors
3. Proposition: A Parametrization Is Conformal if and only if It is Isothermal
4. Theorem: Any Two Regular Surfaces Are Locally Conformal

L12_B 1. Proposition: A Criterion for Local Conformal
2. Stereographic Projection

L12_C 1. Stereographic Projection (cont.)

第 13 講 The Intrinsic Geometry of Surfaces: Gauss Remarkable Theorem

L13_A 1. Christoffel Symbols

L13_B 1. Christoffel Symbols (cont.)
2. Christoffel Symbols in Terms of the First Fundamental Form

L13_C 1. All Geometric Concepts and Properties Expressed in Terms of the Christoffel Symbols Are Invariant under Isometry
2. Codazzi-Mainardi Equations

第 14 講 The Intrinsic Geometry of Surfaces: Gauss Remarkable Theorem (cont.)

L23_A 1. Proposition: Theorem of Turning Tangents
2. The Integral of a Differentiable Function over a Bounded Region on an Oriented Surface
3. Theorem: Local Version of Gauss-Bonnet Theorem

L23_B 1. Theorem: Local Version of Gauss-Bonnet Theorem (cont.)

L23_C 1. Theorem: Local Version of Gauss-Bonnet Theorem (cont.)
2. Theorem: Global Gauss-Bonnet Theorem

第 24 講 Gauss-Bonnet Theorem

L24_A 1. Triangulation
2. Euler Characteristic Number
3. Proposition: Every Regular Region of a Regular Surface Admits a Triangulation

L24_B 1. Proof: Global Gauss-Bonnet Theorem

L24_C 1. Proof: Global Gauss-Bonnet Theorem (cont.)

第 25 講 Gauss-Bonnet Theorem (cont.)

L25_A 1. Theorem: Gauss-Bonnet Theorem for Orientable Compact Surface
2. Example: Sphere with Radius r
3. Example: Convex Surface in R^3
4. Example: Polar Cap

L25_B 1. Example: Polar Cap (cont.)
2. Euler Characteristic Number and Genus
3. Theorem: Diffeomorphic Surfaces Have the Same Euler Characteristic Number and Two Compact Oriented Surfaces with the Same Euler Characteristic Number Are Diffeomorphic

L25_C 1. Theorem: A compact Oriented Surface with Positive Gauss Curvature Is Diffeomorphic to a Standard Sphere
2. Four Color Map Theorem

第 26 講 Clairaut's Theorem

L26_A 1. Proposition: A Regular Compact Connected Oriented Surface Which Is Not Homeomorphic to a Sphere Has Some Points Such That the Gauss Curvature Is Positive, Negative and Zero
2. Proposition: Clairaut's Theorem

L26_B 1. Proposition: Clairaut's Theorem (cont.)

L26_C 1. Surface of Revolution and Hyperbolic Models

第 27 講 Hyperbolic Models

L27_A 1. Hyperbolic Models: Pseudo-Sphere, Upper Half-Plane and Poincare Disc

L27_B 1. Geodesics of Upper Half-plane – By Clairaut's Theorem

L27_C 1. Geodesics of Upper Half-plane – By Geodesic Equations

第 28 講 Mobius Transformation and Non-Euclidean Geometry

L28_A 1. Mobius Transformation

2. Mobius Transformation from Upper Half-plane to Upper Half-Plane Is an Isometry

L28_B 1. Mobius Transformation from Upper Half-plane to Upper Half-Plane Is an Isometry (cont.)

2. Five Postulates for Euclidean Geometry

L28_C 1. The Parallel Postulate and Non-Euclidean Geometry