

第 1 講 The Sign of Gauss Curvature

- L1_A** 1. Review: Some Setting in the Last Semester
2. Example: The Surface Given by $z=h(x,y)$ with $K(p)>0$

- L1_B** 1. Example: The Surface Given by $z=h(x,y)$ with $K(p)<0$
2. Example: The Surface Given by $z=h(x,y)$ with $K(p)=0$

- L1_C** 1. Example: Torus and Monkey Saddle

第 2 講 Geometric Interpretation of Gauss Curvature

- L2_A** 1. Orientation Preserving and Orientation Reversing Map
2. Proposition: The Gauss Map is Orientation Preserving at Elliptic Point and Orientation Reversing at Hyperbolic Point

- L2_B** 1. Proposition: Geometric Interpretation of Gauss Curvature

- L2_C** 1. Proposition: Geometric Interpretation of Gauss Curvature (cont.)
2. Examples: Sphere and Trough-Shaped Surface

第 3 講 Local Convex and Curvature

- L3_A** 1. Review: Geometric Interpretation of Gauss Curvature
2. Remark: Similar Result for Curvature of Plane Curve
3. Tangent Indicatrix
4. Locally Convex and Strictly Locally Convex

- L3_B** 1. Note: Relations between Curvature and Locally Convex

- L3_C** 1. Note: A Critical Point of a Distance Function on a Surface

第 4 講 The Rigidity of the Sphere

- L4_A** 1. Theorem: A Compact Surface Has an Elliptic Point

- L4_B** 1. Theorem: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere
2. Lemma: Three Conditions of a Point to Be an Umbilical Point
3. Proof: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere

- L4_C** 1. Proof: A Compact Connected Surface with Constant Gauss Curvature Is a Sphere (cont.)
2. Proof: Three Conditions of a Point to Be an Umbilical Point

第 5 講 The Rigidity of the Sphere (cont.)

- L5_A** 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

- L5_B** 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

- L5_C** 1. Proof: Three Conditions of a Point to Be an Umbilical Point (cont.)

第 6 講 Vector Field

- L6_A** 1. Vector Field
2. Trajectory of a Vector Field
3. Examples: $w=(x,y)$ and $w=(y,-x)$

- L6_B** 1. Theorem: Existence and Uniqueness of the Trajectory of a Vector Field
- 2. Theorem: Existence of the Local Flow a Vector Field

- L6_C** 1. Ruled Surface, Ruling and Directrix
- 2. Examples: Plane, Cylinder, Cone and Hyperboloid of Revolution

第 7 講 **Ruled Surface**

- L7_A** 1. Line of Striction

- L7_B** 1. Condition of a Ruled Surface Given by the Line of Striction as Directrix being a Regular Surface
- 2. Gauss Curvature of a Ruled Surface Given by the Line of Striction as Directrix

- L7_C** 1. Developable Surface
- 2. Developable Surface Has Gauss Curvature Zero at Regular Points

第 8 講 **Developable Surface**

- L8_A** 1. Two Subclasses of Developable surface

- L8_B** 1. Two Subclasses of Developable surface (cont.)
- 2. Example: The Envelope of the Family of Tangent Planes Along a Curve of a Surface

- L8_C** 1. The Envelope of the Family of Tangent Planes Along a Curve of a Surface Is Developable

第 9 講 **Minimal Surface**

- L9_A** 1. Review: Developable Surface
- 2. Minimal Surface
- 3. Normal Variation
- 4. Interpretation of Minimality

- L9_B** 1. Interpretation of Minimality (cont.)

- L9_C** 1. Interpretation of Minimality (cont.)
- 2. Proposition: A Parametrized Surface Is Minimal if and only if $A'(0)=0$

第 10 講 **Minimal Surface (cont.)**

- L10_A** 1. Proof: A Parametrized Surface Is Minimal if and only if $A'(0)=0$
- 2. Isothermal Parametrized Surface
- 3. Theorem: If x Is an Isothermal Parametrized Surface
Then $x_{uu}+x_{vv}=2(a^2)HN$

- L10_B** 1. Theorem: If x Is an Isothermal Parametrized Surface
Then $x_{uu}+x_{vv}=2(a^2)HN$ (cont.)
- 2. Harmonic Function
- 3. Corollary: An Isothermal Parametrized Surface Is Minimal if and only if
Its Coordinate Functions Are Harmonic
- 4. Introduction: Development of Minimal Surface

- L10_C** 1. Examples: Catenoid and Helicoid
- 2. Proposition: Any Minimal Surface of Revolution Is an Open Subset of a Plane or a Catenoid
- 3. Proposition: Any Ruled Minimal Surface Is an Open Subset of a Plane or a Helicoid
- 4. Theorem: There Is No Compact Minimal Surface

第 11 講 **The Intrinsic Geometry of Surfaces: Isometries and Conformal**

- L11_A** 1. Parametrizations for Catenoid and Helicoid
- 2. Isometry and Local Isometry
- 3. Example: Local Isometric; The Cylinder and Plane in \mathbb{R}^2
- L11_B** 1. Example: Local Isometric; The Cylinder and Plane in \mathbb{R}^2 (cont.)
- 2. Example: Every Helicoid Is Locally Isometric to Catenoid
- 3. Proposition: Two Regular Surfaces Have the Same First Fundamental Form in Domain U if and only if They Are Locally Isometric
- L11_C** 1. Proposition: Two Regular Surfaces Have the Same First Fundamental Form in Domain U if and only if They Are Locally Isometric (cont.)
- 2. Conformal

第 12 講 **The Intrinsic Geometry of Surfaces: Isometries and Conformal (cont.)**

- L12_A** 1. Review: Isometry and Conformal
- 2. Note: A Conformal Map Preserves the Angle between Two Tangent Vectors
- 3. Proposition: A Parametrization Is Conformal if and only if It is Isothermal
- 4. Theorem: Any Two Regular Surfaces Are Locally Conformal
- L12_B** 1. Proposition: A Criterion for Local Conformal
- 2. Stereographic Projection
- L12_C** 1. Stereographic Projection (cont.)

第 13 講 **The Intrinsic Geometry of Surfaces: Gauss Remarkable Theorem**

- L13_A** 1. Christoffel Symbols
- L13_B** 1. Christoffel Symbols (cont.)
- 2. Christoffel Symbols in Terms of the First Fundamental Form
- L13_C** 1. All Geometric Concepts and Properties Expressed in Terms of the Christoffel Symbols Are Invariant under Isometry
- 2. Codazzi-Mainardi Equations

第 14 講 **The Intrinsic Geometry of Surfaces: Gauss Remarkable Theorem (cont.)**

L14_A 1. Codazzi-Mainardi Equations and Gauss Formula

L14_B 1. Codazzi-Mainardi Equations and Gauss Formula (cont.)

L14_C 1. Codazzi-Mainardi Equations and Gauss Formula (cont.)
2. Gauss's Theorema Egregium

第 15 講 **The Intrinsic Geometry of Surfaces: Gauss Remarkable Theorem (cont.)**

L15_A 1. Codazzi-Mainardi Equations and Gauss Formula (cont.)

L15_B 1. Gauss Curvature in Terms of the First Fundamental Form
2. Example: Surface of Revolution
3. Proof: Gauss's Theorema Egregium
4. Example: Catenoid and Helicoid

第 16 講 **The Intrinsic Geometry of Surfaces: Fundamental Theorem of Surface**

L16_A 1. Review: Gauss's Theorema Egregium
2. Counterexample: The Converse of Gauss's Theorema Egregium Is Not True

L16_B 1. Counterexample: The Converse of Gauss's Theorema Egregium Is Not True (cont.)
2. Theorem: Fundamental Theorem of Surface (Bonnet)

L16_C 1. Example: Is There a Regular Surface with the Given Differentiable Functions E, F, G, e, f, g

第 17 講 **Parallel Transport and Geodesics**

L17_A 1. Example: Is There a Regular Surface with the Given Differentiable Functions E, F, G, e, f, g (cont.)
2. Covariant Derivative

L17_B 1. General Formula of the Covariant Derivative
2. Example: Covariant Derivate of a Vector Field on a Plane
3. Parallel Vector Field
4. Proposition: There Exists a Unique Parallel Vector Field along a Curve with Given Initial Value

L17_C 1. Proposition: The Inner Product of Two Parallel Vector Fields Is Constant
2. Example: The Tangent Vector Field of a Meridian Is a Parallel Vector Field on a Sphere

第 18 講 **Parallel Transport and Geodesics (cont.)**

L18_A 1. Parallel Transport

L18_B 1. Parameterized Geodesic and Geodesic
2. Algebraic Value and Geodesic Curvature

L18_C 1. Geometric Interpretation of Geodesic Curvature

2. Example: Geodesic Curvature of a Circle on a Unit Sphere

第 19 講 **Algebra Value of the Covariant Derivative**

L19_A 1. Example: The Normal Curvature and the Geodesic Curvature of the Circle on the Elliptic Parabolic

2. Lemma: The Differentiable Extension of a Determination

L19_B 1. Lemma: Relation between the Covariant Derivative of Two Unit Vector Fields and the Variation of the Angle That They Form

2. Note: The Geodesic Curvature Is the Rate of Change of the Angle That the Tangent to the Curve Makes with a Parallel Vector Field

3. Proposition: An Expression for the Algebraic Value in Terms of the First Fundamental Form and the Variation of the Angle

L19_C 1. Proposition: An Expression for the Algebraic Value in Terms of the First Fundamental Form and the Variation of the Angle (cont.)

第 20 講 **Algebra Value of the Covariant Derivative (cont.)**

L20_A 1. Proposition: Liouville's Formula

L20_B 1. Proposition: Liouville's Formula (cont.)

2. Geodesic Equations

L20_C 1. Geometric Interpretation of Geodesic

第 21 講 **Geodesic Equations**

L21_A 1. Example: Geodesics of a Cylinder

2. Example: Geodesics of a Surface of Revolution

L21_B 1. Example: Geodesics of a Surface of Revolution (cont.)

L21_C 1. Example: Geodesics of a Sphere

2. Geodesic Parametrization and Geodesic Coordinates

第 22 講 **Surfaces of constant Gaussian curvature**

L22_A 1. Review: Geodesic Parametrization and Geodesic Coordinates

2. Theorem: Any Point of a Surface of Constant Gauss Curvature Is Contained in a Coordinates Neighborhood That Is Isometric to an Open Set of a Plane, a Sphere or a Pseudo-Sphere

L22_B 1. Theorem: Any Point of a Surface of Constant Gauss Curvature Is Contained in a Coordinates Neighborhood That Is Isometric to an Open Set of a Plane, a Sphere or a Pseudo-Sphere (cont.)

2. Simple Closed Piecewise Regular Parametrized Curve

L22_C 1. Closed Vertices and Regular Arcs

2. Differentiable Functions That Measure the Positive Angle from x_u to the Tangent of a Simple Closed Curve

第 23 講 **Gauss-Bonnet Theorem for Simple Closed Curves and Curvilinear Polygons**

- L23_A** 1. Proposition: Theorem of Turning Tangents
- 2. The Integral of a Differentiable Function over a Bounded Region on an Oriented Surface
- 3. Theorem: Local Version of Gauss-Bonnet Theorem

L23_B 1. Theorem: Local Version of Gauss-Bonnet Theorem (cont.)

- L23_C** 1. Theorem: Local Version of Gauss-Bonnet Theorem (cont.)
- 2. Theorem: Global Gauss-Bonnet Theorem

第 24 講 **Gauss-Bonnet Theorem**

- L24_A** 1. Triangulation
- 2. Euler Characteristic Number
- 3. Proposition: Every Regular Region of a Regular Surface Admits a Triangulation

L24_B 1. Proof: Global Gauss-Bonnet Theorem

L24_C 1. Proof: Global Gauss-Bonnet Theorem (cont.)

第 25 講 **Gauss-Bonnet Theorem (cont.)**

- L25_A** 1. Theorem: Gauss-Bonnet Theorem for Orientable Compact Surface
- 2. Example: Sphere with Radius r
- 3. Example: Convex Surface in \mathbb{R}^3
- 4. Example: Polar Cap

- L25_B** 1. Example: Polar Cap (cont.)
- 2. Euler Characteristic Number and Genus
- 3. Theorem: Diffeomorphic Surfaces Have the Same Euler Characteristic Number and Two Compact Oriented Surfaces with the Same Euler Characteristic Number Are Diffeomorphic

- L25_C** 1. Theorem: A compact Oriented Surface with Positive Gauss Curvature Is Diffeomorphic to a Standard Sphere
- 2. Four Color Map Theorem

第 26 講 **Clairaut's Theorem**

- L26_A** 1. Proposition: A Regular Compact Connected Oriented Surface Which Is Not Homeomorphic to a Sphere Has Some Points Such That the Gauss Curvature Is Positive, Negative and Zero
- 2. Proposition: Clairaut's Theorem

L26_B 1. Proposition: Clairaut's Theorem (cont.)

L26_C 1. Surface of Revolution and Hyperbolic Models

第 27 講 **Hyperbolic Models**

L27_A 1. Hyperbolic Models: Pseudo-Sphere, Upper Half-Plane and Poincare Disc

L27_B 1. Geodesics of Upper Half-plane – By Clairaut's Theorem

L27_C 1. Geodesics of Upper Half-plane – By Geodesic Equations

第 28 講 **Mobius Transformation and Non-Euclidean Geometry**

- L28_A**
 - 1. Mobius Transformation
 - 2. Mobius Transformation from Upper Half-plane to Upper Half-Plane Is an Isometry
- L28_B**
 - 1. Mobius Transformation from Upper Half-plane to Upper Half-Plane Is an Isometry (cont.)
 - 2. Five Postulates for Euclidean Geometry
- L28_C**
 - 1. The Parallel Postulate and Non-Euclidean Geometry